Utilization of Green Roof for Storing Rainfall Water in Urban Areas

Wanda KOWALSKA, Edyta DROŻDŻAL

Cracow University of Technology
Institute of Water Engineering and Water Management
ul. Warszawska 24
31-155 Kraków
tel. +48 12 6282840
wkowalska@smok.wis.pk.edu.pl
edrozdzal@smok.wis.pk.edu.pl

English summary

A mathematical model describing individual processes that take place after rainfall occurs on the surface of so called green roof is presented in the paper. The method for solving model equations is given. The advantage of tracing the course of the processes gives the basis for answering the question of how green roofs meet their expected storage capacity demands. An example of model application is presented. Calculations were made for real data and the results were discussed.

Résumé : *Utiliser les toits verts en milieu urbain pour stocker les eaux de pluie*

L'article présente un modèle mathématique décrivant le processus particulier après les précipitations la surface appelé « le toit vert ». La méthode pour solutionner les équations du modèle est proposée. L'avantage de tracer le cours des processus donne la base pour répondre à la question de la façon dont "les toits verts" satisfont leurs besoins prévus de capacité de stockage. Un exemple de l'application du modèle est présenté. Des calculs ont été effectués pour de vraies données et les résultats ont été discutés.

Streszczenie polskie: *Zastosowanie « zielonego dachu » w retencjonowaniu opadu w obszarach liejskich*

W referacie przedstawiony został matematyczny model zielonego dachu, opisującego poszczególne procesy zachodzące na nim po pojawieniu się opadu. Podano metodę rozwiązania równań modelu. Uzyskując możliwość śledzenia przebiegu wspomnianych procesów w czasie, stworzono podstawy do odpowiedzi na pytanie, w jakim stopniu zielone dachy spełniają oczekiwaną od nich retencyjną rolę. W referacie podany jest przykład użycia prezentowanego modelu. Przeprowadzono obliczenia dla konkretnych danych i przeanalizowano wyniki

1. Introduction

Decreased storage capacity of river catchments is one out of many essential causes of flood event intensification. This process happens particularly in urban areas. Sealing of vast areas of such surfaces results in the fact that rainwater infiltrates the soil. A considerable part of this amount of water is conveyed through the sewerage system into the river. During intensive rainfalls, fast water outflow generates flood hazard to the areas below the outlet of the sewerage system. Also, as sewers are frequently overfilled, local flooding of buildings and streets occurs frequently.

From among the effective methods for increasing water storage capacity at the place where rainfall occurs, cultivating various plants on terraces, balconies and building roofs is a method possible to apply for larger sets of buildings. This type structures are called green roofs. Interception and evaporation during rainfall causes that a part of water amount is stopped and the rest is delayed in its way to the surrounding ground or is drained into the sewerage system. Apart from its storage properties, the green cover has also aesthetic values and allows to maintain better air humidity and attenuates the noise. The green roofs in the area of residence or place of employment ensure relax and rest.

Such structures are built mainly in Western countries, including Germany, where they are of park and recreational character. In Poland, despite their high costs and lack of national guidelines and design standards, the possibility of constructing green roofs is considered more and more frequently.

2. Basic constructional components of green roof

The green roof is a multi-layers structure (Fig.1). Its basic components are:

- waterproof layer,
- thermal insulation,
- draining layer
- vegetation layer.

The waterproof layer should be resistant to failures caused by plant roots, and to chemicals, e.g., fertilizers, and also be completely resistant biologically to mildew and fungi. It should be resistant to microorganisms and humus acids.

The draining layer serves the purposes of constant and full water draining from the vegetation layer and roof surface. The layer is usually made from washed gravel of the 8/16 mm grain or fine aggregate, e.g. keramsite, which has good draining properties.

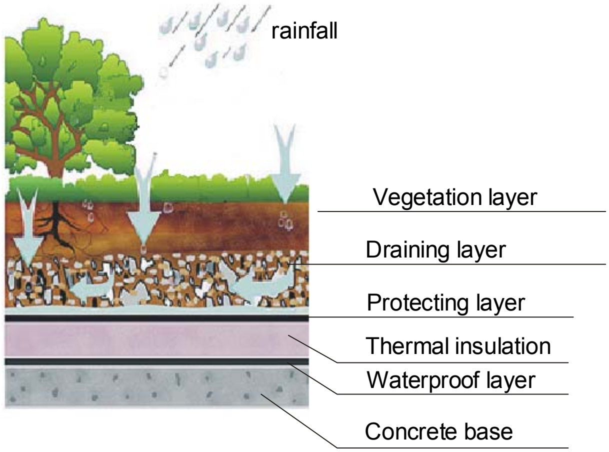

Fig.1. Green roof layers

Water storing mats with hollows with waterproof walls (Fig. 2) are also utilized. Excess of water collected in the hollows is drained by a system of channels in between the external walls of the hollows.

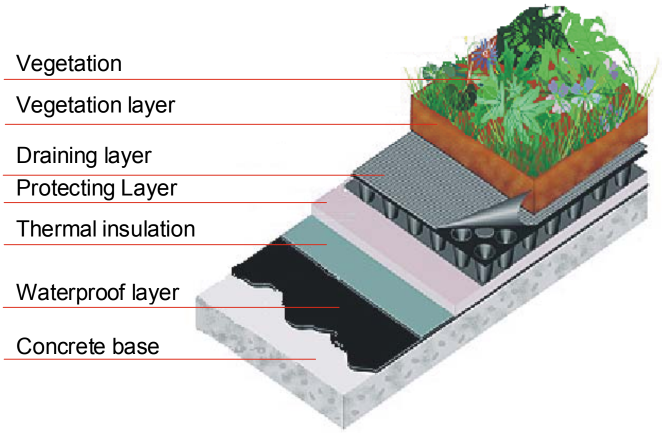

Fig.2. Green roof components

The protecting layer isolates the draining layer from the waterproof layer or – depending on the roof construction system – from the thermal insulation layer. The protecting layer (interlayer) is usually made of a polypropylene geotextile. In most cases, the vegetation layer consists of humus mixed with a material of mineral origin. The grain size of this layer should ensure it the stable structure, which enables water excess to be released to the draining layer. The vegetation layer thickness should range from 9 to 15 cm for extensive cultivation, to 35 cm for intensive low cultivation, and from 35 to 250 cm for intensive high cultivation. The vegetation and draining layers are separated by a filtration layer from a geotextile preventing small soil particles from entering the draining layer, protecting it against silting.

As it was mentioned above, taking into consideration the type of vegetation used to green roof construction, two types of green roof can be distinguished:

- intensive green roofs,
- extensive green roofs.

For extensive cultivation, low plants are used with low demands. This plants, e.g. steppe plants, can live and grow with no special care.

Intensive cultivation means roof gardens with lawns, bushes and trees. The plants used there have high demands concerning their care and require regular feeding with water and nutrients.

The choice of the type of vegetation cover depends, first of all, on the roof stability and strength, and also on its tilt. Extensively cultivated green roofs are most frequent as their construction and maintenance are not too expensive. If such a vegetation cover is used, the green roof construction is thin-layered. Intensive cultivation requires careful selection of material for individual layers; also plant watering should be then planned.

3. Mathematical model of rainwater seepage through the green roof layers

To describe hydrological processes occurring on a green roof after rainfall occurrence, the amounts of interception and evapotranspiration are determined, then net rainfall is transformed into runoff. The latter process was described with two models.

A. Mathematical model of rainwater seepage in the vegetation layer

Formulating the model, an assumption was accepted that water movement in the vegetation layer is an unsteady one-dimensional vertical movement. Flow parameters depend on time *t* and distance *z*. Rain intensity course in time *t* is represented by rain intensity values determined for specified time intervals Δt (Fig.3). The Green-Ampt model was employed to describe water movement. It is adopted here that each change in rain intensity $\Delta nd_i = nd_i - nd_{i-1}$, $i = 1,2,...il$ generates a new, *i*-th wetting front of constant moisture Θ_i in the area between the soil surface and the first location of the generated front (see Fig.4). All wetting fronts generated in successive time intervals travel down the vegetation layer with a constant velocity vf_i, characteristic for a given front, reaching locations $zf_i^{(j)}$ at successive time instants $t^{(j)} = 1,2,...,kj$. The process continues until the water collected in the soil volume bounded from below by the wetting front is drained from the vegetation layer. In time intervals with constant rain intensity (interval $\left[t^{(0)}, t^{(3)}\right]$ in Fig.3), single rain intensity change (in Fig.3: Δnd_1 at time instant $t^{(0)}$) generates one wetting front.

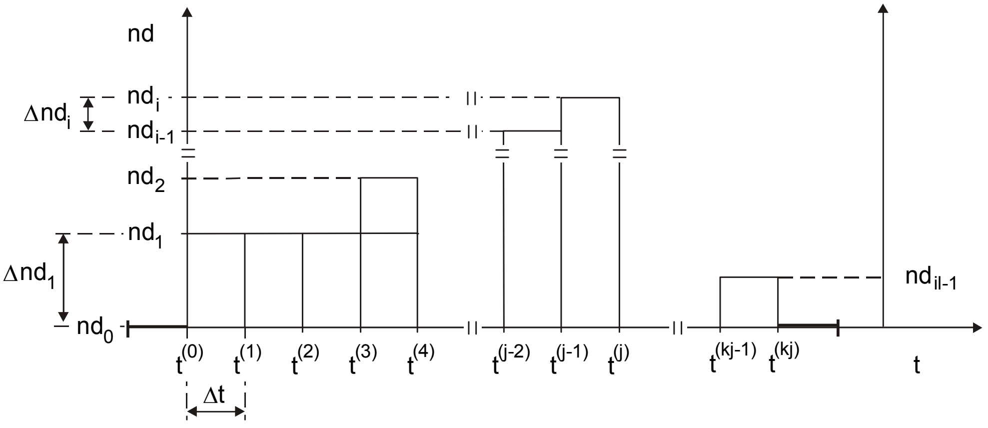

Fig.3. Hyetograph

To determine soil feed with rainwater and soil moisture Θ_i in the area above the *i*-th front generated during the time from $t^{(i-1)}$ to $t^{(0)}$, rainfall intensity nd_i existing at that time is compared with the soil infiltration capacity expressed by the following formula:

$$vinf_i^{(j)} = kf(\Theta_i) \cdot I_i^{(j)} \qquad (1)$$

where $kf(\Theta_i)$ denotes the soil water hydraulic conductivity, and $I_{(i)}^{(j)}$ hydraulic gradient. These quantities are described by the following formulas:

$$kf(\Theta_i) = kn \cdot \left(\frac{\Theta_i - \Theta o}{\Theta n - \Theta o} \right)^{\alpha} \qquad (2)$$

$$I_{(i)}^{(j)} = \begin{cases} 1 + \dfrac{hw^{(j)} + hk}{zf_i^{(j)}} & for \quad \Theta_i = \Theta n \\ 1 & for \quad \Theta_i < \Theta n \end{cases} \qquad (3)$$

where:

α – soil-type dependent parameter, [–]; $\alpha \in [3;4]$

kn – coefficient of permeability, [*m*/*s*]

$hw^{(j)}$ – depth of ponding at time $t^{(j)}$, [*m*]

hk – capillary rise in the vegetation layer, [*m*]

Θ_i – the moisture in the area between soil surface and *i*-th front, [–]

Θn – maximum soil moisture, [–];
referring to full saturation ($\Theta n \cong 0.85\,n$),

Θo – permanent wilting point, [–]

$zf_i^{(j)}$ – *i*-th front range (from soil surface) at time instant $t^{(j)}$

If rainfall intensity nd_i during the time interval from $t^{(j-1)}$ to $t^{(j)}$ is less than kn, soil feed rate during *i*-th front generation equals rainfall intensity. In this case the value of Θ_i is calculated using equation (2) after nd_i is put into the left side of (2). The resulting soil moisture Θ_i is less than Θn. In this case the soil will not be saturated with water.

If rainfall intensity exceeds kn value, soil moisture reaches its maximum value equal to Θn.

If rainfall intensity exceeds kn and is less than the soil infiltration capacity $vinf_i^{(j)}$, soil feeding rate during *i*-th front generation equals rainfall intensity nd_i. When rainfall intensity exceeds the soil infiltration capacity, soil is fed with the rate of $vinf_i^{(j)}$. In this case, the difference between the rainfall intensity and soil feeding rate will generate a change $\Delta hw^{(j)} = hw^{(j)} - hw^{(j-1)}$ of water layer depth on soil surface, expressed by the formula:

$$\Delta hw^{(j)} = (nd_i - vinf_i^{(j)}) \cdot \Delta t \qquad (4)$$

In order to determine the *i*-th front travel velocity, $vf^{(j)}$, a value of $\Delta V^{(j)}$ is calculated, being a soil water volume change per unit area of the cross-section of the area under consideration in time interval from $t^{(j-1)}$ to $t^{(j)}$ when the *i*-th front was generated. As it was mentioned above, this volume change was generated by a rainfall intensity change $\Delta nd_i = nd_i - nd_{i-1}$, so the following equality holds:

$$\Delta V^{(j)} = (nd_i - nd_{i-1}) \cdot \Delta t \qquad (5)$$

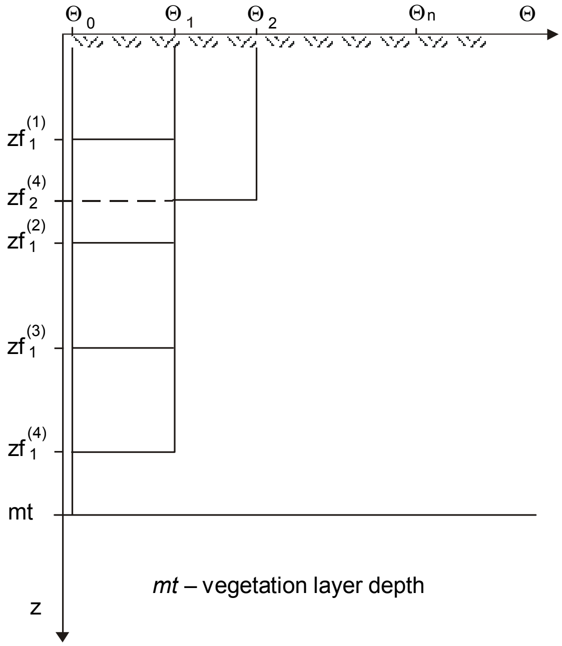

Fig.4. Generation of constant-moisture fronts in soil

After time interval from $t^{(j-1)}$ to $t^{(j)}$ elapses, the newly generated front will reach the depth of $zf_i^{(j)}$, and soil moisture above it will be Θ_i. The previous moisture of this area was Θ_{i-1}, so the soil water volume changed by $\Delta V^{(j)}$ (see Fig. 4) expressed by the formula

$$\Delta V^{(j)} = (\Theta_i - \Theta_{i-1}) \cdot zf_i^{(j)} \tag{6}$$

Comparing equations (5) and (6), the formula for front range is obtained:

$$zf_i^{(j)} = \frac{(nd_i - nd_{i-1}) \cdot \Delta t}{\Theta_i - \Theta_{i-1}} \tag{7}$$

Dividing both sides of equation (7) by Δt and replacing the resulting left-side ratio by velocity vf_i gives the following formula for front travel velocity:

$$vf_i = \frac{nd_i - nd_{i-1}}{\Theta_i - \Theta_{i-1}}. \tag{8}$$

At the time the *i*-th front traveling down the vegetation layer reaches its bottom, water outflow will occur at rate specified by equation (2). The outflow rate will change after next front reaches the bottom of the layer.

B. Mathematical model of rainwater seepage in the draining layer

To determine unsteady water flow in the draining layer being the saturation zone, Boussinesq equation, two-dimensional in horizontal plane, was adopted:

$$\frac{\partial}{\partial x}(h \cdot kn \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y}(h \cdot kn \frac{\partial H}{\partial y}) + w = \mu \frac{\partial H}{\partial t} \tag{9}$$

where:

μ – storage coefficient (effective porosity), [-]
w – infiltration recharge (from vegetation layer), [*m*/*s*]
h – hydraulic depth of the aquifer, [*m*]
kn – coefficient of water permeability, [*m*/*s*]
H **–** hydraulic head, [*m*].

The equation above is a nonlinear second-order partial parabolic differential equation. To obtain a unique solution of equation (9), it is necessary to specify the initial condition and boundary conditions. Together with adopted conditions, equation (9) constitutes an initial-boundary problem with unique solution.

The domain where the solution of equation (9) is sought is a rectangle $Lx \times Ly$ with its sides measured along the *x* and *y* axes, respectively, of the Cartesian co-ordinate system with its origin at the lower left vertex of the mentioned rectangle (see Fig. 5). This rectangle is the base of a rectangular prism cut of the green roof, whose three lateral faces are impermeable while in the middle of the fourth face, at its base, a rectangular outflow opening of width Lw and height hw is located. The rectangular prism is filled with gravel up to the height, mz , overlaid with the vegetation layer.

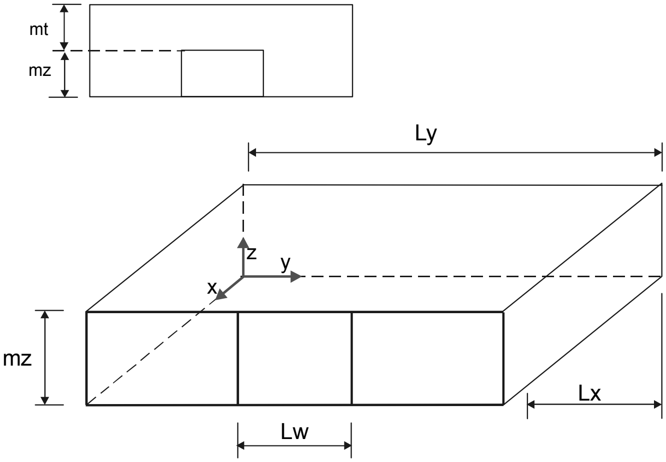

Fig.5. A green roof section

Initial condition adopted was:

$$h(x, y, 0) = 0 \tag{10}$$

Boundary conditions for rectangle sides (except for a section of this side to which the outflow opening adjoins) are given as zero water flow velocity component perpendicular to the rectangle sides:

$$\begin{aligned} & v_y(x, y, t) = 0 \quad for \quad x \in [0, Lx], y = 0 \;\; or \\ & y = Ly, \;\; t \geq 0, \end{aligned} \tag{11}$$

$$\begin{aligned} & v_x(x, y, t) = 0 \quad for \;\; x = 0, \qquad y \in [0, Ly], \;\; t \geq 0 \\ & and \quad for \;\; x = Lx, \quad y \in [0, (Ly - Lw)/2)] \cup [(Ly + Lw)/2, Ly]. \end{aligned} \tag{12}$$

For the section of the rectangle side to which the outflow opening adjoins, the boundary condition is assumed as the critical depth:

$$H(Lx, y, t) = hkr \quad for \;\; y \in [(Ly - Lw)/2, (Ly + Lw)/2] \tag{13}$$

The above initial-boundary problem was solved using finite difference method with the five-point explicit scheme presented in Fig. 7.

Applying the described models and solution methods with initial and boundary conditions, a relevant computer code was developed. The calculation results allow to observe the course of rainwater flow through the green roof layers enabling an assessment to be made of how the storage demand of the green roof is met.

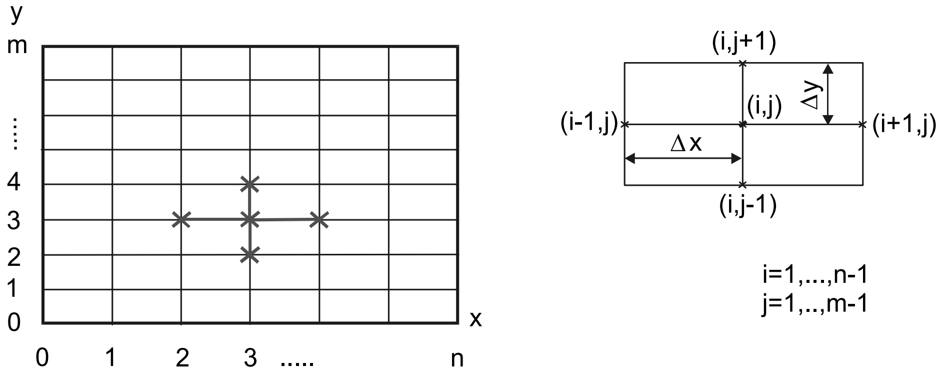

Fig.6. Discretisation of the calculation domain

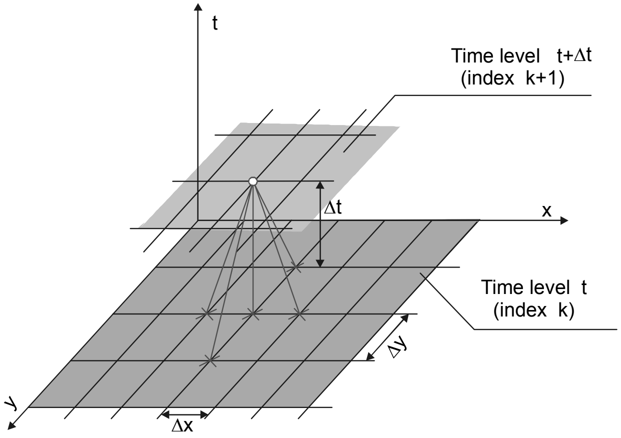

Fig.7. Five-point explicit difference scheme

4. Calculation example

Four calculation examples were considered. In the first one, it was assumed that the roof was covered with sheet metal only, in the second and third, a gravel layer on the roof was taken into account, in the fourth case rainwater flow through vegetation and draining layers on the roof was considered.

It was assumed that the area of rainwater flow within the roof surface is of rectangular prism shape divided along roof length into sections of identical dimensions. Carrying out calculations for the first case, it was assumed that rainwater flows down from the roof on the whole its width. One-dimensional kinematic wave equation was used to model the surface flow. Two ways of water draining from a gravel-covered roof were analyzed: along all the roof width (case 2) or through the opening located in the middle part of the front wall of each section (see Fig. 5), at its bottom base (case 3). The rainwater flow over the roof surface was in both cases described by the Boussinesq equation, one-dimensional in case 2, two-dimensional in case 3. When calculating rainwater flow through vegetation and draining layers of the green roof (case 4), mathematical model described in detail in Section 3 was applied assuming as in case 3 that water is drained off the roof by the opening located in the middle part of the front wall of each section.

All calculations were made for individual roof section of $10\,m \times 4.45\,m \times 0.19\,m$. The obtained results are repeatable for each section. The width and height of the opening in the middle part of the front wall of the section were $0.15\,m$ and $0.1\,m$, respectively. When gravel layer was considered on the roof, it was assumed that this was a homogeneous layer of constant thickness $mz = 0.1\,m$ with coefficient of water permeability $kn = 100\ m/day$. If over the gravel layer being a draining layer a vegetation layer was located, it was assumed that its thickness $mt = 0.09\,m$, it was made of peat with coefficients kn, α, n are 0.15, 3.5 and 0.85, respectively. Calculations were made for peat initial moisture equal to the plant permanent wilting point Θo.

Calculation for an individual roof section were carried out on a rectangular net spread over the roof with $\Delta x \times \Delta y = 0.25m \times 0.05m$ mesh. Time step Δt = 60s was used in calculation made for the vegetation layer while the time step assumed for the sheet-metal roof and gravel layer was smaller (Δt = 6s) which was caused by the stability requirements of the used difference schemes.

In the first three calculation cases, it was assumed the rainfall intensity $n(t)$ is constant and its duration is as in Fig. 8. The results of calculations made for these cases are shown in Figs. 9 through 14 as input rainwater hydrographs and the hydrographs of rainwater released from the roof, and as time courses of input and output water volumes.

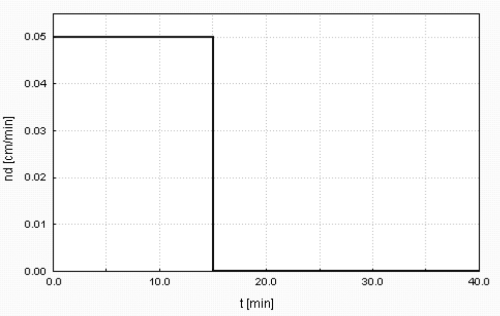

Fig. 8. Hyetograph used in the example

Analysis of the results for the roof without any soil layer (Fig. 9 and Fig.10) makes it possible to state that as early as after 15 minutes from the beginning of the rainfall of 15-minute duration 92% of rainwater flowed from the roof, and all rainfall volume was released after 25 minutes.

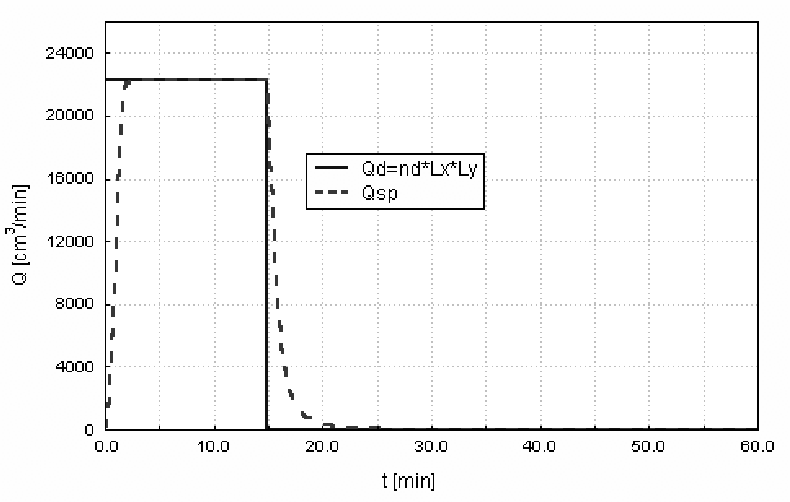

Fig.9. Input and output rainwater hydrographs for the sheet-metal roof

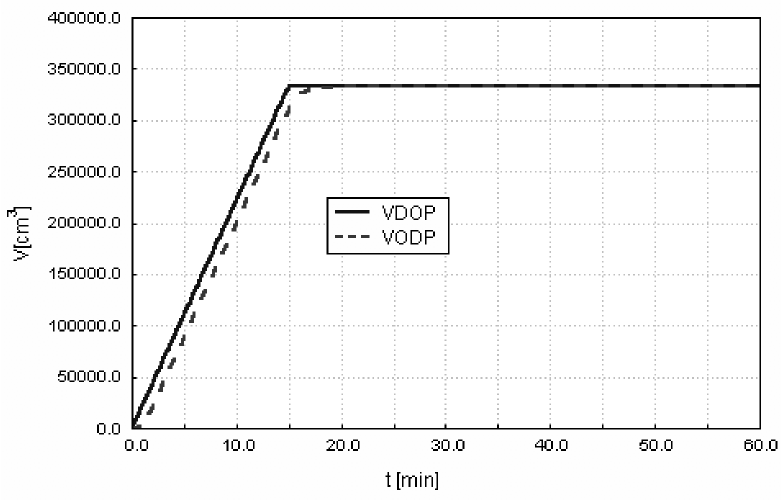

Fig. 10. Time course of input and output rainwater volume for the sheet-metal roof

In Figs. 11 and 12 the results are presented of calculations made for the roof covered with a gravel layer, from which water flows along all the width of the roof. Fig. 12 shows that after 300 minutes, about 32% of volume of water that fell onto the roof is drained from it.

In every cases the right scale concerns break curve on figure.

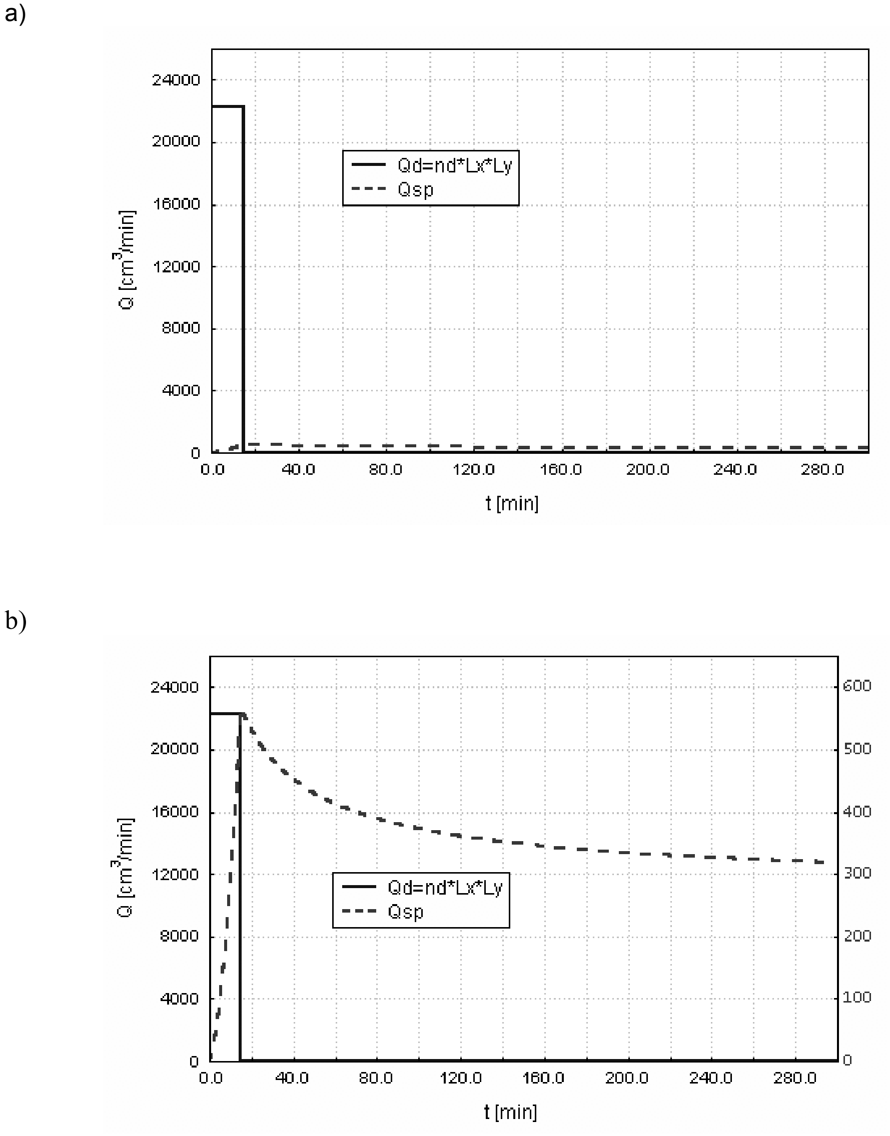

Fig.11. Input and output rainwater hydrographs for the roof covered with a gravel layer (runoff along all the width of the roof); a) real scale, b) magnified scale for Q_{sp}

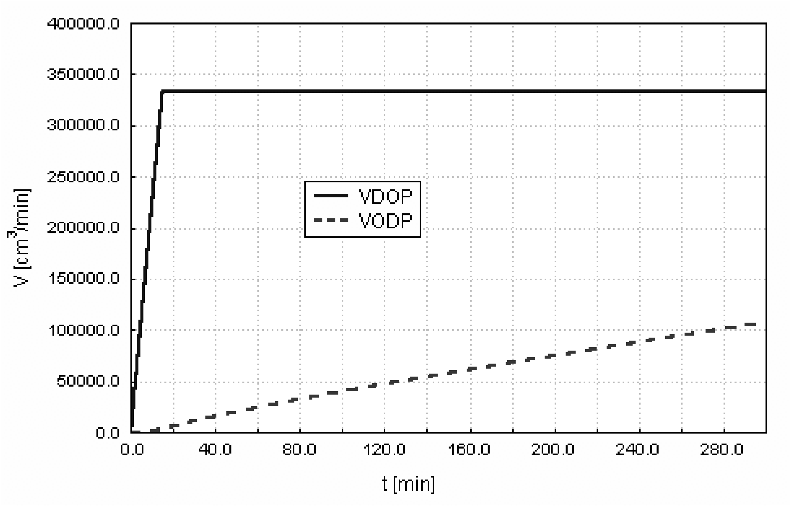

Fig. 12. Time course of input and output rainwater volume for the roof covered with a gravel layer (runoff along all the width of the roof)

Figs. 13 and 14 refer to the cases when rainwater flow from the roof covered with a gravel layer takes place through a 15-cm wide opening in the middle of the roof section wall. As calculations show, only 2% of rainwater accumulated on the roof was drained after 300 minutes.

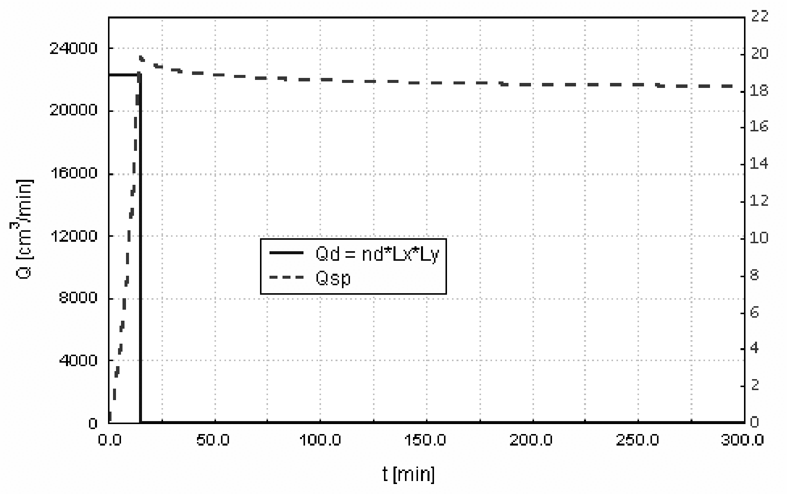

Fig.13. Input and output rainwater hydrographs for the roof covered with a gravel layer (runoff through the opening)

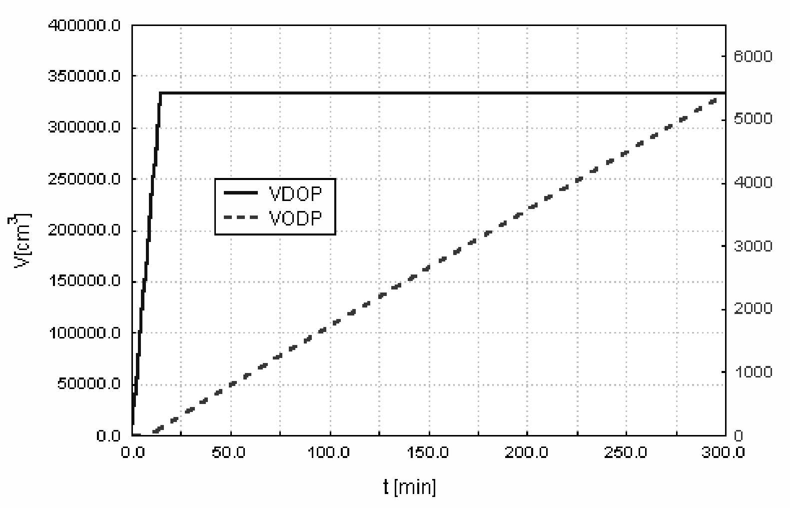

Fig. 14. Time course of input and output rainwater volume for the roof covered with a gravel layer (runoff through the opening)

Seven successive figures refer to the case when rainwater flows through the layers constituting the green roof (case 4). In Figs. 15 and 16 a hyetograph assumed for calculations is presented together with the resulting rainwater inflow hydrograph. The assumption of constant-intensity rainfall was made for simplify result analysis. Figs. 17 and 18 show the rainwater traveling through the peat layer, in the form of a water outflow velocity time course and a hydrograph of outflow from this layer. Next Figs. 19 and 20 show hydrograph of rainwater outflow from the roof and time course of rainwater volume flowing into the draining layer and leaving the green roof. Fig. 20 shows that after 300 minutes from the beginning of rainfall, so after 170 minutes from the commencement of draining layer feed, only 0.2% of water reaching that layer was drained from the green roof.

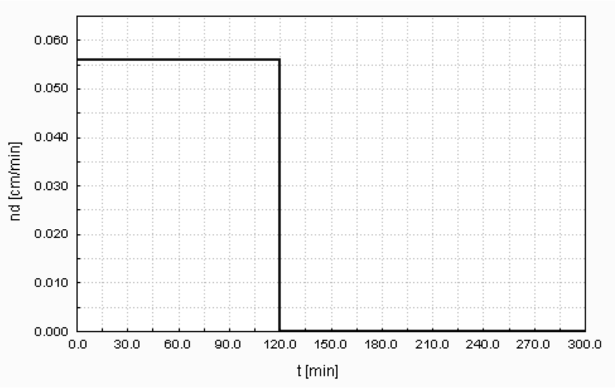

Fig. 15. Hyetograph assumed for calculations of water runoff through the green roof layers

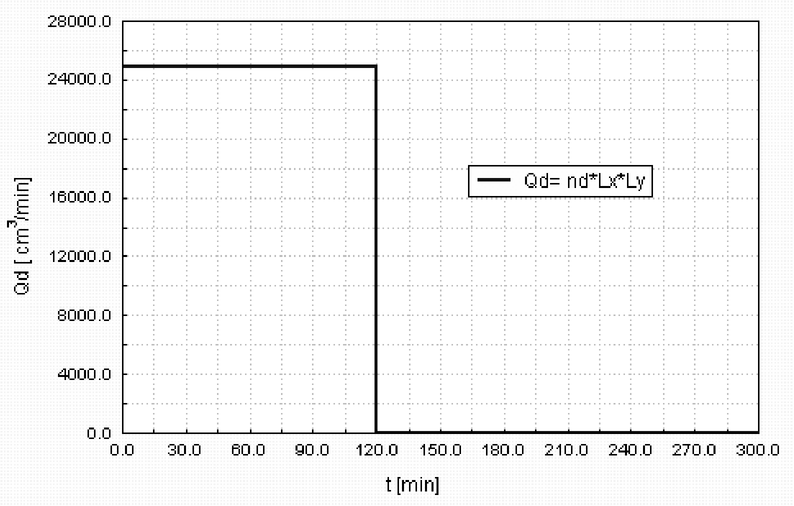

Fig.16. Hydrograph of rainwater inflow onto the surface of the green roof

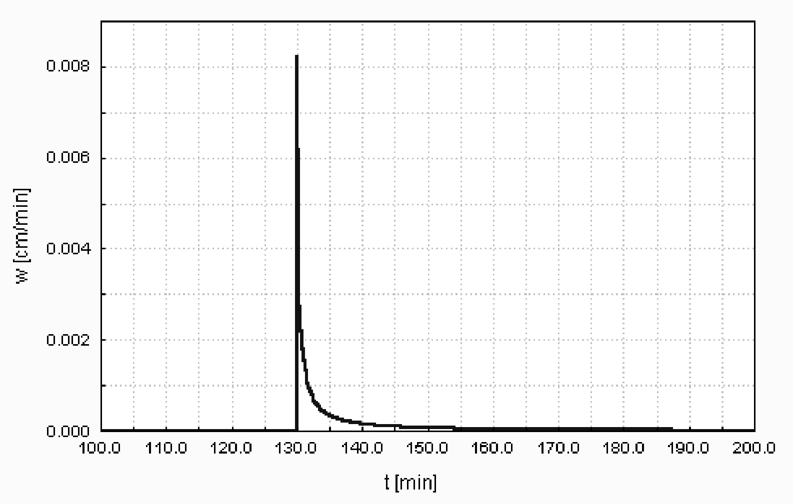

Fig.17. Time course of velocity of rainwater outflow from the green roof vegetation layer

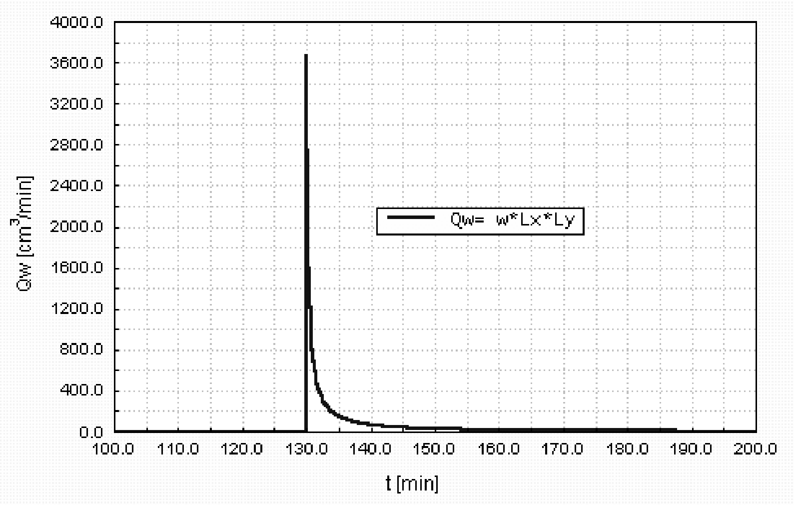

Fig.18. Hydrograph of rainwater outflow from the green roof vegetation layer (equivalent to the hydrograph of inflow to the draining layer)

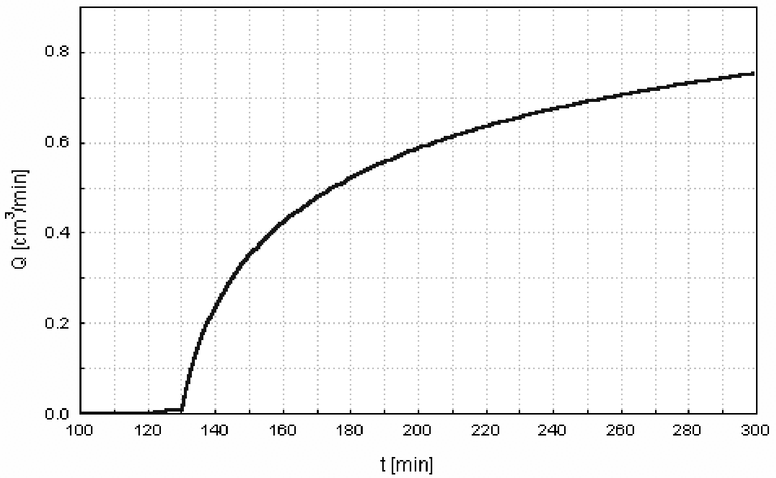

Fig.19. Hydrograph of rainwater outflow from the green roof (after passing through the vegetation and draining layers)

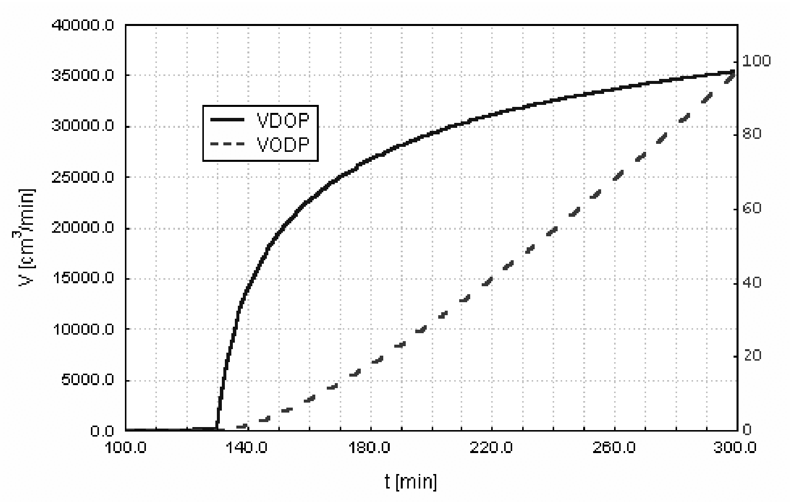

Fig.20. Volume time course of rainwater flowing into the draining layer and out of the green roof

Calculation results presented in Fig. 21 show temporal variability of water volume falling onto the roof and drained from it. After passing through both layers, the volume Vodp = 100 cm^3 of water flowed off. This volume is about 0.003 % of the total rainwater volume.

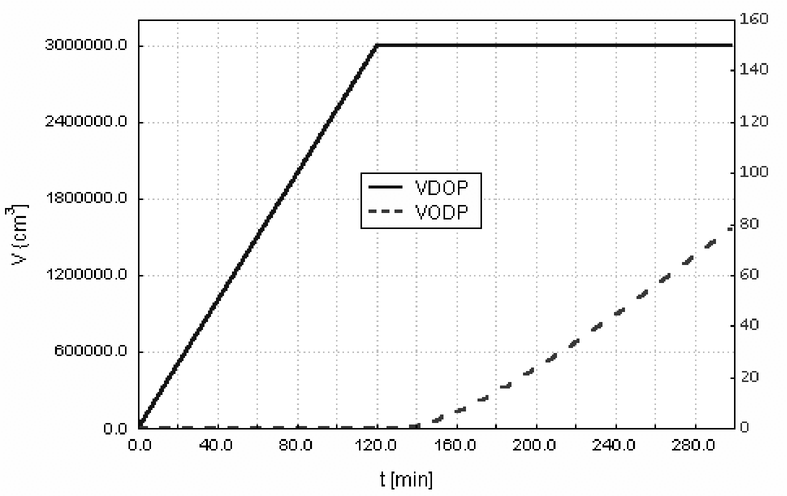

Fig.21. Volume time course of rainwater falling onto the green roof and flowing out of the green roof

5. Concluding remarks

It can be stated that a green roof of construction described in this paper meets effectively the expected storage demands. Such a structure intercepts precipitation water and substantially lengthens the time of its outflow, reducing at the same time to some extent water volume. The results of the calculation example presented above are a good illustration of this statement. It should also be stressed that the basis for the calculations done above included an innovative numerical model of green roof; it seems that it is both reliable and effective model.

It is certain that further research work is purposeful, both as the work on theoretical description of functioning of green roofs, and the work on design solutions. This results from the experience of implementations made, assessed with changing climatic conditions and dynamic progress of urbanization taken into account, which frequently generates flood hazard. A green roof is a structure that, if widely utilized in urban areas, can substantially mitigate this hazard.

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