A one dimensional sediment transport model and its application to a mountainous river Raba

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English summary

The aim of the paper is to present a numerical simulation of sediment transport in a mountainous river. Two one-dimensional sediment transport and riverbed evolution models Rubarbe and Metoda are used for predicting the variation of longitudinal bed profile along the river reach and changes in the cross-sectional geometry due to erosion or deposition of sediment.
In the current paper, simulation of the sediment transport is applied to the Raba River – a mountain tributary of the Wisla River. The simulation of riverbed evolution is applied for the reach located from 81.829 km to 77.751 km (upstream from the Dobczyce Dam), about 20 km to the south of Krakow. The results of the simulation from the two models are analysed and discussed.

Résumé français : *Un modèle unidimensionnel de transport de sédiments et son application à une rivière de montagne, la Raba.*

Le but de cette communication est de présenter les simulations numériques du transport de sédiments dans une rivière de montagne. Deux modèles unidimensionnels simulant le transport de sédiments et l'évolution du lit, les modèles RubarBE et Metoda sont utilisés pour prédire la variation du profil en long du lit et les changements de la géométrie des profils en travers dus à l'érosion ou au dépôt de sédiments.
Dans notre communication, ces modèles sont appliqués à la Raba, un affluent de la Vistule. Le bief étudié s'étend du PK 81,829 au PK 77,751 (à l'amont du barrage Dobczyce), 20 kilomètres environ au Sud de Cracovie. Les résultats des calculs par les deux modèles sont analysés et discutés.

Streszczenie polskie : *Zastosowanie jednowymiarowego modelu transportu rumowiska w symulacji procesów korytotwórczych rzeki górskiej*

W artykule przedstawiono symulację przepływu rumowiska w korycie rzeki górskiej przy użyciu jednowymiarowego modelu matematycznego. Zaprezentowane zostały dwa modele: Rubarbe oraz Metoda, dotyczące modelowania zmian występujących w korytach cieków w wyniku zachodzących procesów erozji i sedymentacji.
Modele Rubarbe i Metoda zostały zastosowane do modelowania przepływu rumowiska w korycie rzeki Raby – karpackiego dopływu Wisły. Obliczenia przeprowadzono dla odcinka od 81.829 km do 77.751 km, zlokalizowanego powyżej zbiornika w Dobczycach. Odcinek ten znajduje się w sąsiedztwie drogi Kraków – Zakopane. Obliczenia wykonano za pomocą modeli Rubarbe i Metody i przeprowadzono analizę wyników.

1. Presentation of the Raba River

The Raba river, in southern Poland, is a mountain tributary of the Wisła River. In this region, the topography of drainage catchments is highly varied. The division of the Carpathian rivers courses is as follows: the upper course (with the average slopes: 7 ‰ – 15 ‰), the middle and the lower courses (with the average slopes 2.5 ‰ – 0.5 ‰).

In the Carpathian rivers a high variety of water stages is observed: a rapid-growing flow appears especially in spring and early summer while, in the period of dry season or during the long-lasting snow-cover on rivers, a low-flow period can be observed. In the Raba river, during the flood, discharge can reach the value Q = 800 m^3/s in the upper course, and about Q = 1500 m^3/s in the lower course.
The total catchment area of the Raba River is 1537.10 km^2. At Km 60.00 of the River course, the Dobczyce dam is located.

The Raba River is characterized by an erosion and deposition process which occurs with varied intensity along the river course. However, the main erosion process is observed downstream from the Dobczyce dam (Matyas and Łapuszek, 2000).

2.Presentation of the experimental reach

In the current paper, simulation of the sediment transport and riverbed evolution is applied for the reach which is located (Fig. 2) from 81.829 km to 77.751 km (upstream from the Dobczyce Dam). The experimental reach is located about 30 km to the south of Krakow. The experimental reach is closed to the important road Kraków – Zakopane. The river in the presented reach was repeatedly straightened and narrowed during the 20th century. The hydraulic engineering activity is the most important factor of the strong erosion of riverbed in the mentioned reach of the Raba River (Ratomski and Witowska, 1993). The observations show that up to 3 metres of bed degradation has occurred since the beginning of the engineering activity (Fig. 1).

A few years ago a project of extension of the international road situated next to the river Raba was set up. Leading to a partial replacement and narrowing of the riverbed along the road is applied (see table 1, Fig. 3).
The engineering activity on the experimental reach started in spring 2003. Work is due to be completed at the end of October 2003. At this point morphology of the evolution of the Raba River will be examined.

Parameter	Value
The catchment area [m^2]	642
Width of the riverbed before works [m]	45 - 100
Width of the riverbed after works [m]	40 - 78
Variation of slope before works [-]	0.0014 – 0.0064
Variation of slope after works [-]	0.003
Nominal diameter [mm]	28
Diameter of active layer [mm]	78

Table 1. Parameters of the experimental reach.

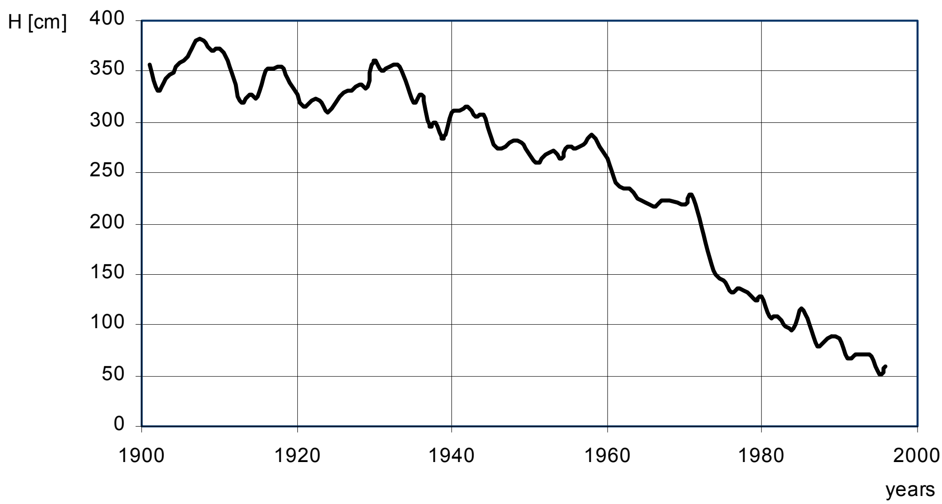

Fig. 1 The riverbed erosion intensity on the experimental reach of the Raba River.

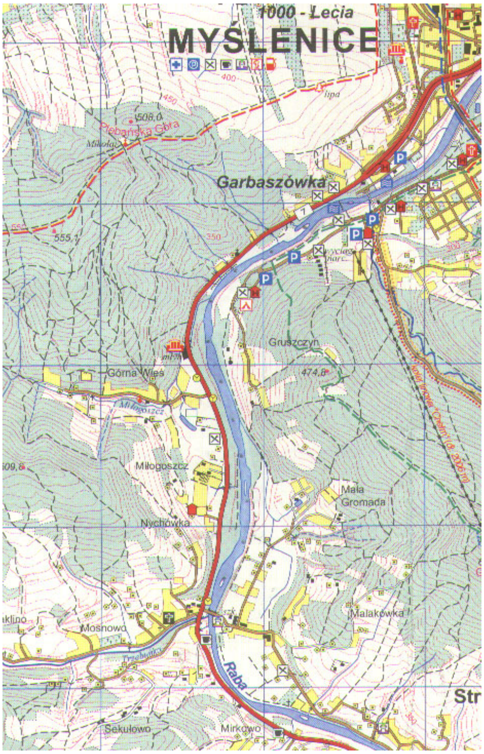

Fig. 2 The experimental Raba River reach: 81.581 km – 77.751 km.

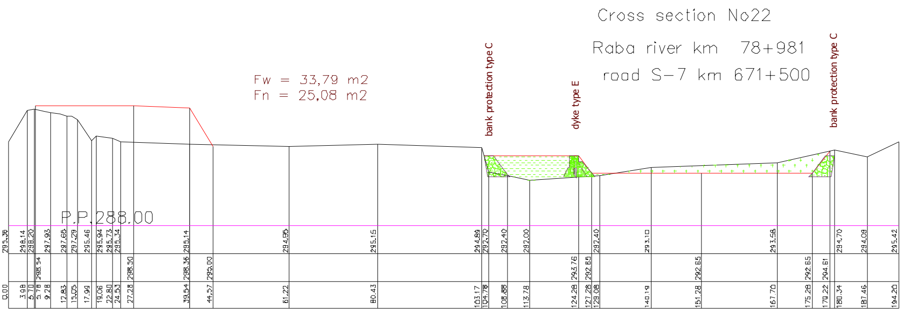

Fig. 3 Example of cross-section with the modifications of the project

3. The RubarBE and METODA models

The aim of this paper is to present the numerical simulation of sediment transport in the mountainous river. Two one-dimensional sediment transport and riverbed evolution models are used: METODA and RubarBE. 1-D model: RUBARBE has been developed by Cemagref; it is described in full in the article “What are the problems to be solved by a 1-D river sediment transport model ? Example of RubarBE software” (Paquier, 2003).

3.1.Description of the METODA model

The one - dimensional model METODA was used for predicting variation of longitudinal riverbed profile along the Raba River reach, and for predicting changes in the cross-sectional geometry. This model describes the changes in river morphology due to erosion or deposition of sediment (Piwowarczyk Ogorek, 2003), (Piwowarczyk Ogorek et al., 2000).
The model is based on the system of the following equations:

- The Saint Venant equations for water:

(1) $$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q_b$$

(2) $$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_2}{A}\right)+gA\frac{\partial z}{\partial x}+gAS_f+q_b u=0$$

where:
A – cross-sectional flow area [m^2], g – acceleration due to gravity [m/s^2], Q –water discharge [m^3/s], R – hydraulic radius [m], q_b – lateral water flow [m^3/s]; z – water surface elevation [m]; t – time [s]; x – streamwise coordinate [m]; S_f – slope of the energy line [-]: $S_f=\frac{Q^2}{K^2}$, where: $K=\frac{1}{n}AR^{2/3}$

- Conservation of bed-material:

(3) $$\frac{\partial z}{\partial t}+\beta_1\frac{1}{\gamma_r B}\frac{\partial G_r}{\partial x}+\beta_2\frac{1}{\gamma_u B}\frac{\partial G_u}{\partial x}=0$$

where:
B – cross-sectional width [m]; G_r – sediment discharge [N/s]; G_u – suspended load discharge [N/s]; z - bottom elevation [m], β - the coefficients of quantity of movement, x – streamwise coordinate [m].
The assumption is that the movement of suspended load is omitted.

• Meyer-Peter and Muller formula for sediment transport:

$$\gamma \frac{Q_s}{Q}\left(\frac{k_s}{k_r}\right)^{3/2} h S_f = 0.047\gamma_s d + 0.25\left(\frac{\gamma}{g}\right)^{1/3} q_s^{2/3} \tag{4}$$

where:
q_s – sediment flow [N·s^{-1}m^{-1}]; k_s, k_r – velocity coefficient in Manning Strickler formulae [-]; h – water depth; d - sediment diameter; S_f – slope of the energy line [-], Q – water discharge [m^3/s], Q_s - water discharge due to beginning of sediment movement [m^3/s], γ_r – specific weight of sediment, γ_r=2650 [kg/m^3], g – acceleration due to gravity [m/s^2].

In the model presented, the systems of equations are solved separately for each time period. Fig.4 shows details of the schematic representation of longitudinal profile and cross-sections and Fig.5 shows the scheme of METODA model.

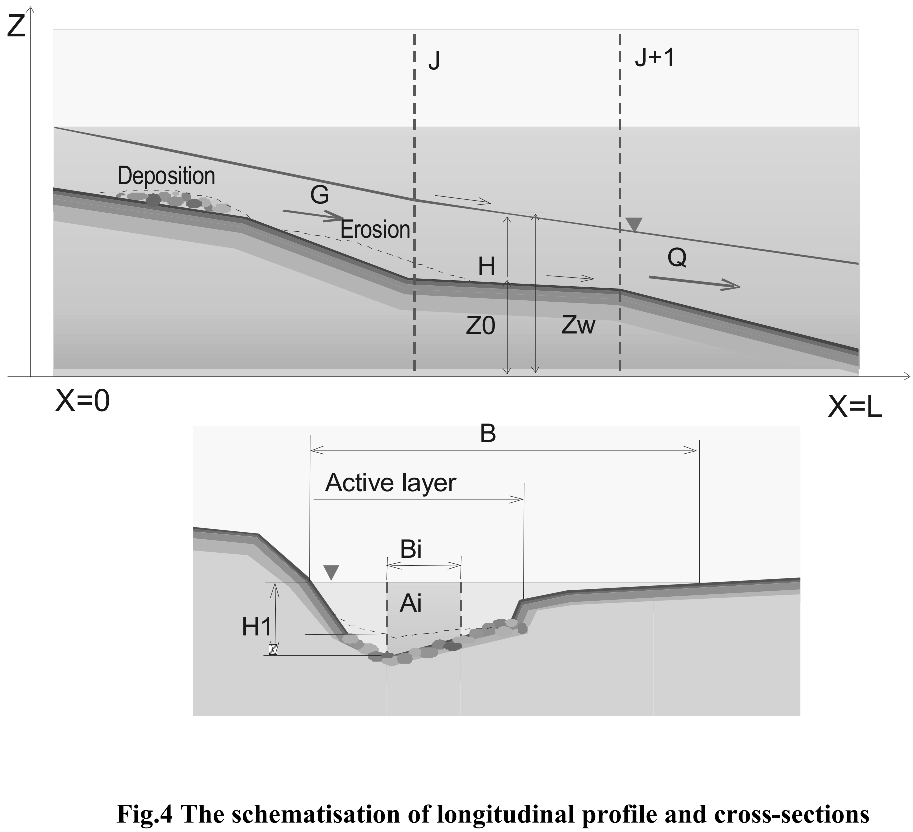

Fig.4 The schematisation of longitudinal profile and cross-sections

where: Zw – water surface coordinate, Zo – riverbed level, h – water level, B – width of free water surface, Bi – width of *i* – band, Ai – *i* – band area.

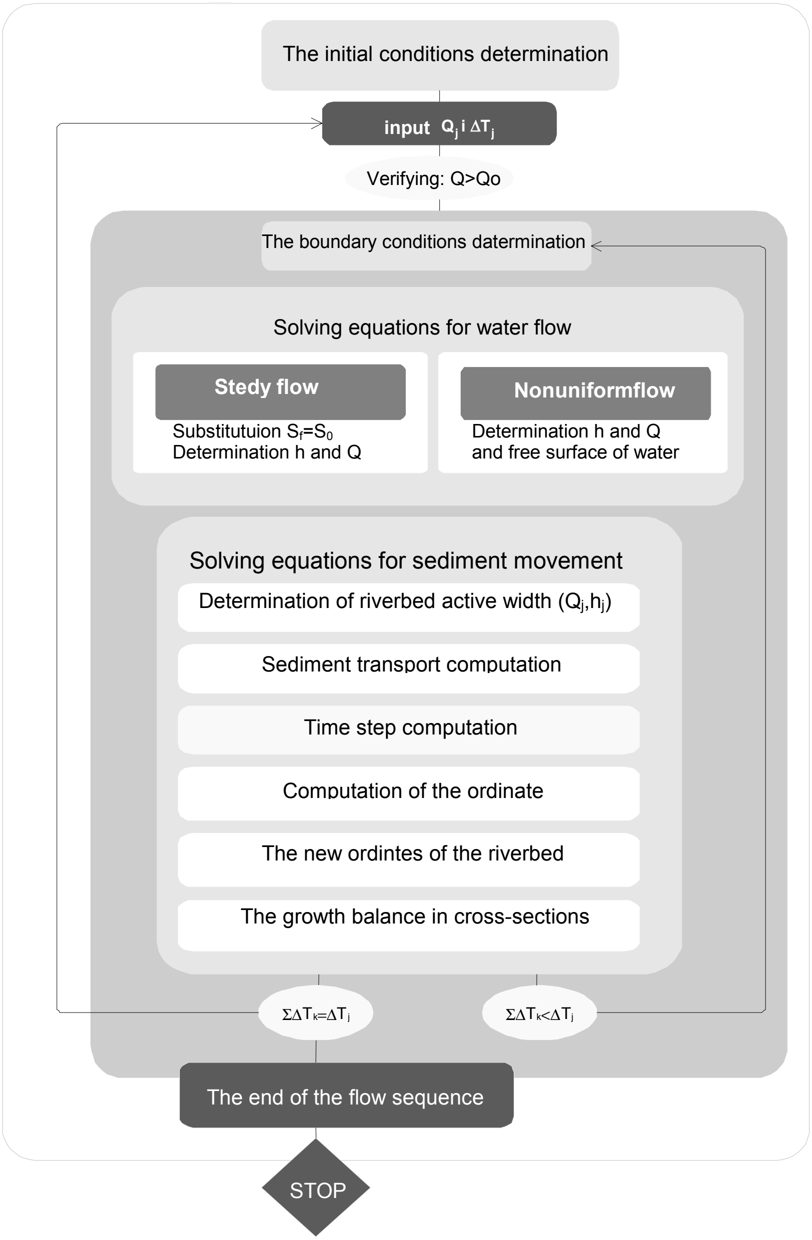

Fig. 5 The block diagram of the METODA model.

It is assumed in the model that the study reach is in dynamic equilibrium and only bed load transport is considered. The equation of mass conservation of sediment is solved by an explicit numerical scheme:

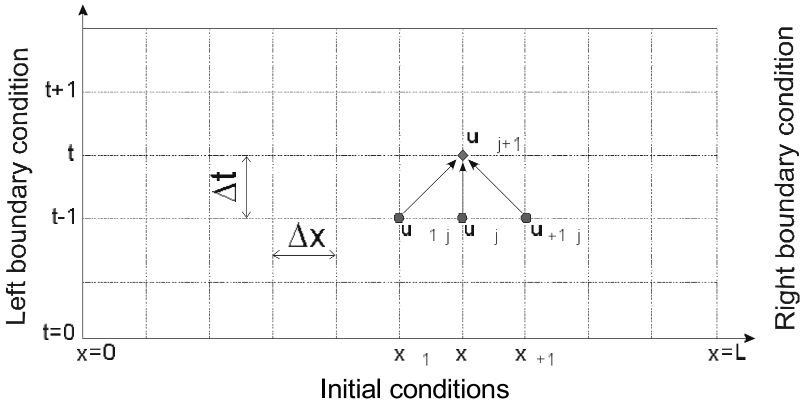

Fig. 6 Computation grid.

(5) $$\frac{\Delta z_{i-1,i}}{\Delta t}=\frac{1}{1-p}\frac{q_{si}^{j}-q_{si-1}^{j}}{\Delta x}$$

(6) $$\frac{\Delta z_{i,i+1}}{\Delta t}=\frac{1}{1-p}\frac{q_{si+1}^{j}-q_{si}^{j}}{\Delta x}$$

where:

$\Delta z_{i,I+1}$ - change in bed elevation, q_{si}^{j} - sediment transport in unit of width, t – time, x – distance along channel axis, z_i^j – bed level at cross-section i at time j, Δt and Δx respectively the time step and the space step used in the discretisation, p – porosity of sediments.

The new channel cross-section z_i^j at station i, which is used at the next time iteration $j+1$ is determined by adding the bed elevation change to the previous bed elevation (time j). The bed elevation change at station i (positive for aggradation, negative for scour) is obtained by:

(7) $$\Delta z_i=z_i^{j+1}-z_i^{j}$$

(8) $$\Delta z=\frac{\Delta z_{i-1,i}\cdot\Delta x_{i-1,i}+\Delta z_{i,i+1}\cdot\Delta x_{i,i+1}}{\Delta x_{i-1,i}+\Delta x_{i,i+1}}$$

In the case of natural cross-section, the geometry is not usually homogenous. The model computes changes in the riverbed at each band of cross-section. Riverbed elevations are an average of each node between adjacent bands.

4.The study cases

In order to compare both models, the simulation of riverbed deformation is done as follows:
- discharge Q=250 m^3/s with the duration t=12 hours,
- flood discharge hydrograph (Fig. 7) chosen from data set of 30 years (1971 – 1999).

44 prismatic cross-sectional channel geometry are used by the models RubarBE and METODA. The initial conditions for sediment transport is Q_s=0 kg/s, and the sediment boundary conditions is Q_s=0 kg/s. The downstream conditions for the experimental reach (km 77.751) are fixed by the rating curve (9) for the dam which is located there:

(9) $$f(Q) = -0.000007Q^2 + 0.0082Q + 0.9615$$

where: Q is water discharge [m^3/s].
The mean diameter is D_{50}=28 mm and the standard deviation is σ=1. On the base on value D_{50}, a Manning Strickler coefficient is 34.6 for the mean section of the study reach, and 14.3 for the flood plain.
The simulation is also done for 19 cross-sections of the same reach and with the same initial and boundary conditions.

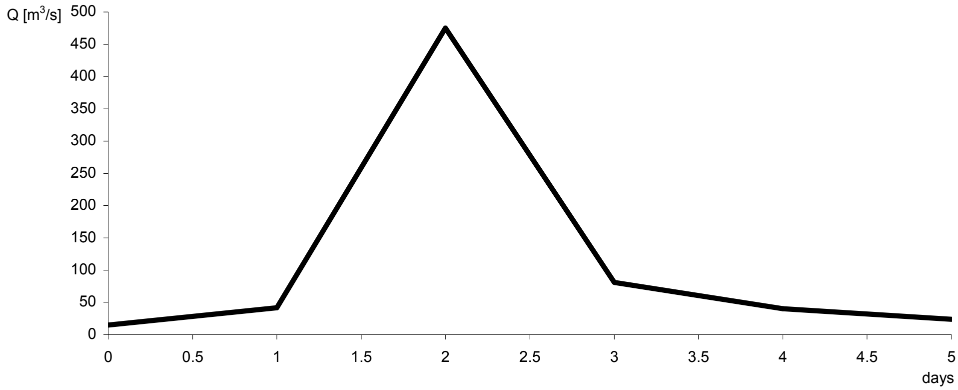

Fig. 7 Flood discharge hydrograph.

In order to have a full simulation of the riverbed evolution, a computation will be done for the more complete flow sequence, that means, for 30 years.
The most important part of the computation will consist of analysing the impact of the project execution on the riverbed evolution. The next computations will simulate sediment transport in the experimental reach after the river training. The comparison of these two models will be performed with a steady flow of 250 m^3/s and the flood hydrograph.

5.The computational results analysis

The first comparison of the two models is performed for the computation carried out for steady flow *Q* = 250 m^3/s, with the total simulation time of 12 hours. The results show that erosion and deposition are located in the same computational cross-sections with the same order of magnitude (Fig. 8).

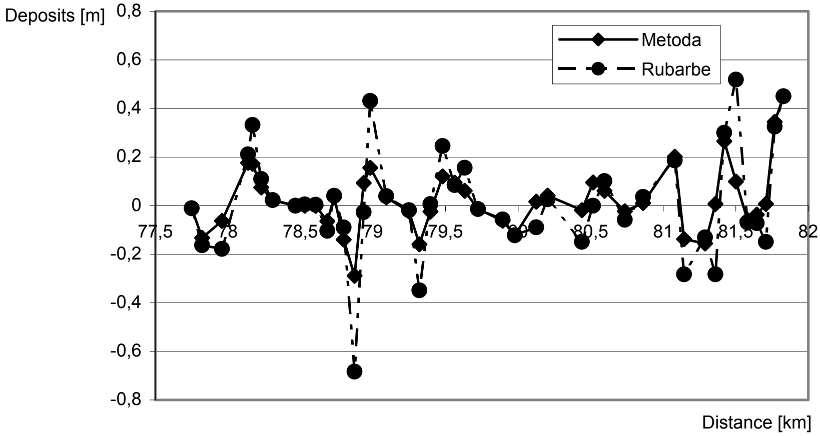

Figure 8. Bottom level change for steady flow Q = 250 m^3/s

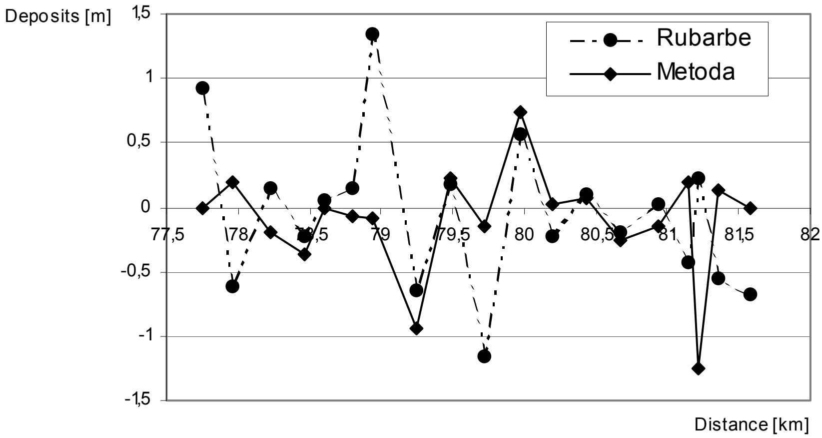

Figure 9. Bottom level change for the discharge hydrograph

The second comparison is performed for the flood hydrograph (Fig. 9) and for 19 cross-sections of the same experimental reach. The results of both models are different, in the upstream and downstream of the reach. The reason for these differences might be caused by the fact that the upstream and downstream boundary conditions in the two models were not exactly the same. Moreover, the