Dynamic Modelling of Fruit Tree – Inertia Shaker System

Zoltán Láng

Technical Department, Corvinus University, Budapest. Villányi u. 31, 1118 Budapest, Hungary; e-mail: zoltan.lang@uni-corvinus.hu

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Abstract

Idle and in charge power equations of a shaker with rotating eccentric–weight and of a slider crank type one were set up and compared. A tree structure model was built of trunk and main roots. Using its static equations the location of virtual turning centre versus shaking height function was defined. Based on this function a simple three element model composed of reduced mass, spring constant and kinetic damping coefficient was transferred from a trunk cross-section to all others. Replacing these reduced parameters into the efficient power equation of the two types of shaker and into the equation of trunk amplitude the efficient power consumption and amplitude versus shaking height curves were drawn. These functions, with the free combination of eccentricity, shaking frequency and mass parameters of the shakers help to optimize machine design. Calculating the relation efficient power consumption/trunk amplitude for all trunk cross sections the most efficient clamping heights of the shakers were found.

INTRODUCTION

Modelling of fruit trees looks back to the same history as the experiments with the first shaker harvester. In the differential equation of the fruit tree-shaker system of Fridley and Adrian (1966) the tree was replaced by a three-element model, which was vibrated by a sinusoidal changing force, generated by unbalance-slugs (Fig. 1.):

\[ M \ddot{x}_M + k \dot{x}_M + \frac{1}{c} x_M = m r \omega^2 \sin \omega t \]

(1)

The momentary power input for a rotating type shaker as shown in Fig. 1 is then:

\[ P_r = \frac{m r \omega^3 X}{2} [\sin(2 \omega t - \varphi) + \sin \varphi] \]

(2)

For the calculation of the trunk displacement amplitude \(X\) the following well known equation was used:

\[ X = \frac{m r \omega^2}{\sqrt{\left(\frac{1}{c} - M_j \omega^2\right)^2 + (k \omega)^2}} \]

(3)

For the phase angle: \(\tan \varphi = \frac{k \cdot \omega \cdot c}{1 - M_j \cdot c \cdot \omega^2} \)

(4)

With the assumption that the shaking frequency is much higher, than the fundamental mode frequency of the limb (\(\omega >> \omega_n\)) the calculation of the limb peak-to-peak stroke (\(S\)
was getting very simple: it depended on the masses of the system and of the eccentricity of unbalanced masses:

\[
S \equiv \frac{2mr}{M_t}
\]  

(5)

This widely used Equation 5 of Fridley and Adrian (1966) however doesn’t take into account the elasticity and the damping property of the tree, neither of the shaking frequency.

An attempt to include the above parameters in the model was made by Whitney at al. (1990). The reduced mass, elasticity and the viscous damping coefficient were measured individually on a wooden post fixed to the ground as a vertical cantilever. Comparing measured and calculated data they found that the post acted nearly as a pure spring at the frequencies employed. It means a trunk itself is rather elastic; a great part of the input energy during shaking harvest is absorbed elsewhere.

Horváth and Sitkei (2001) presumed that during shaking the input energy is mostly absorbed in the soil through the rooting system. This means that the trunk translates and turns during shaking and vibrates a certain amount of soul around. Shaking the trunk in different heights they measured the translations and calculated the virtual centre of turning of the tree. They found that the location of this centre changes with the height of shaking, and so does the reduced mass measured at the clamping points. Evaluating run-out acceleration curves of a trunk shaker they defined the logarithmic decrements for different trunk cross-sections (Horváth and Sitkei, 2002).

Láng (2002) measured the translation, rotation and bending of real trunks applying different static loads. As a result of those virtual centres of turning were defined, which changed with the force applied. Apparent static spring constant and viscous damping coefficient of the whole trunk and also of root samples was measured and defined.

The aim of the investigations described below is to describe mathematically the power consumption, generated amplitude and specific power for each trunk cross section to be able to design shaker harvesters more precisely.

MATERIALS AND METHODS

1. Power requirement of inertia shaker

Equation 5 of Fridley and Adrian (1966) for the peak-to-peak stroke of the limb doesn’t differ from the stroke of a mass \( M + M_M \) to which an unbalanced rotating or periodically translating mass \( m \) is clamped (Láng, 2004). In these cases the total mass

\[
M_t = M + m + M_M
\]  

(6)

includes the mass of the shaker body, the exiting mass and the reduced tree mass (Fig. 1).
The theoretical power consumption of a rotating eccentric–weight and of a slider crank shaker mechanism, coupled to a one degree of freedom mass $M_M$ was compared by Láng (2004). Due to the somewhat different working principle of the two units the power equations are different.

For the shaker with rotating eccentric–mass (Fig. 2/a):

$$P_{rth} = \frac{m^2 \cdot r^2 \cdot \omega^3 \cdot \sin 2\omega t}{2 \cdot (M + M_M + m)} \tag{7}$$

For the slider crank type one (Fig. 2/b):

$$P_{sth} = \frac{m \cdot (M + M_M) \cdot r^2 \cdot \omega^3 \cdot \sin 2\omega t}{2 \cdot (M + M_M + m)} \tag{8}$$

Note, that in these equations the limb is regarded as a pure reduced mass; elasticity and damping are neglected. According Fridley and Adrian (1966), Equation 5, consequently Equation 7 gave no exact results all around the limb.

Equations 7 and 8 however can be used to define the idle power consumption of the two types of shakers. In these cases $M_M = 0$. It is interesting to compare the tendencies for the two cases: if $M \to \infty$, than $P_{rth} \to 0$, but $P_{sth} \to (m \cdot r^2 \cdot \omega^3 \cdot \sin 2\omega t) / 2 \tag{9}$

It means increasing the shaker body mass the idle power consumption for the rotating eccentric–weight type shaker decreases, for the slider crank type shaker it tends to the value in Equation 9.

Substituting $M_M$, $c$ and $k$ data measured on real trees into the equation of the three-element model coupled with inertia type shakers (Equation 1, Fig. 1) more realistic trunk
displacement and power requirement data may be achieved for a given trunk cross-
section.

The momentary displacement of the limb in this case is:
\[ x_M = X \cdot \sin(\alpha x - \varphi) \]  
(10)
The momentary velocity of it:
\[ \dot{x}_M = X \cdot \omega \cdot \cos(\alpha x - \varphi) \]  
(11)
and the acceleration:
\[ \ddot{x}_M = -X \cdot \omega^2 \cdot \sin(\alpha x - \varphi) \]  
(12)
For \( X \), the limb displacement amplitude Equation 3 gives the values for both rotating and alternating type shaker.

The average effective power needed to drive a rotating type shaker can be calculated as follows. The energy input for a cycle (Fridley and Adrian, 1966):
\[ W_r = \int P_r \cdot dt = \int_0^{\tau_\varphi} \frac{m r \omega^3 X}{2} \sin(2\alpha x - \varphi) \cdot dt + \int_0^{\tau_\varphi} \frac{m r \omega^3 X}{2} \sin \varphi \cdot dt = \frac{m r \omega^3 X}{2} \sin \varphi \cdot T \] 
(13)
The average effective driving power:
\[ P_{rav} = \frac{W_r}{T} = \frac{m r \omega^3 X}{2} \sin \varphi \]  
(14)
The same equation applies for the slider crank type shaker.

2.2 A new tree structure model
For a simple tree model, built of trunk and main roots (Fig. 3) the following equations can be written:
\[ F = F_{Ax1} + F_{Bx2} - F_{Ax2} - F_{Bx1} = \frac{X}{c} \]  
(15)
\[ F \cdot y = b \cdot (F_{Ay} + F_{By}) + h \cdot (F_{Ax1} + F_{Bx1}) - h \cdot (F_{Ax2} + F_{Bx2}) \]  
(16)
Presuming that \( F_{Ay} = F_{By} \), \( F_{Ax1} = F_{Bx1} \) and \( F_{Ax2} = F_{Bx2} \)
\[ 2 \cdot h \cdot (F_{Ax1} - F_{Ax2}) = F \cdot y - 2 \cdot b \cdot F_{Ay} \]  
(17)
\[ F_{Ax1} - F_{Ax2} = \frac{F \cdot y - 2 \cdot b \cdot F_{Ay}}{2 \cdot h} \]  
(18)
\[ 2 \cdot b \cdot F_{Ay} = F \cdot (y + h) \]  
(19)
and from 18 and 19:
\[ F_{Ax1} - F_{Ax2} = \frac{F}{2} \]  
(20)
Let $x$ be the translation of $O$. Presuming that $c_1 = c_2 = c_0$, than

$$x = \frac{c_0 \cdot F}{2}$$

(21)

The turning angle around $O$:

$$\alpha \equiv \frac{F \cdot c_0}{b} = \frac{F \cdot (y + h) \cdot c_0}{2 \cdot b^2}$$

(22)

Finally the vertical distance of the virtual turning centre $C$ from $O$ will be:

$$\rho \equiv \frac{x}{\alpha} = \frac{c_0 \cdot F}{2 \cdot b} = \frac{b^2}{y + h}$$

(23)

It can be seen that besides the geometrical sizes $h$ and $b$ of the main roots, $\rho$ depends on the vertical position $y$ of the force $F$.

The change of $\rho$ in function of $y$ ($0 < y < 1100$ mm) is shown on Fig. 4, together with the measured $\rho$ values ($\bullet$) of Horváth and Sitkei (2001). The best fitting parameters for the calculation of $\rho$ were as follows: $h = 100$ mm, $b = 680$ mm.
With the help of Equation 23 the reduced trunk mass, spring constant and the viscous damping of a cross section can be reduced to any other.

First let transfer a defined reduced mass $M_{\text{red}}$ of the tree trunk cross-section $y_{\text{red}}$ to $A$ and $B$ (Fig. 5). Let $\rho_{\text{def}}$ be the calculated value of $\rho$ belonging to $y_{\text{def}}$. Keeping the kinetic energy unchanged:

$$\frac{1}{2} M_{\text{def}} \cdot (y_{\text{def}} + \rho_{\text{def}})^2 \cdot \dot{\alpha}^2 = \frac{1}{2} M_{\text{red}} \cdot \left[ (\rho_{\text{def}} - h)^2 + b^2 \right] \cdot \dot{\alpha}^2$$

(24)

$$M_{\text{red}} = \frac{(y_{\text{def}} + \rho)^2}{(\rho_{\text{def}} - h)^2 + b^2} \cdot M_{\text{def}}$$

(25)

Now for any cross-section of the trunk the reduced mass $M(y)$ can be calculated the following way:

$$\frac{1}{2} M(y) \cdot (y + \rho)^2 \cdot \dot{\alpha}^2 = \frac{1}{2} M_{\text{red}} \cdot \left[ (\rho - h)^2 + b^2 \right] \cdot \dot{\alpha}^2$$

(26)

$$M(y) = \frac{(y_{\text{def}} + \rho_{\text{def}})^2}{(y + \rho)^2} \cdot \frac{(\rho - h)^2 + b^2}{(\rho_{\text{def}} - h)^2 + b^2} \cdot M_{\text{def}}$$

(27)

Fig. 6 shows the change of calculated reduced masses along the trunk. The parameters for the calculation were as follows: $y_{\text{def}} = 800$ mm, $\rho_{\text{def}} = 463$ mm, $h = 100$ mm, $b = 680$ mm.

To be able to check the new model’s ability the value measured by Horváth and Sitkei (2001) was replaced into Equation 27 (Fig. 6).

The energy stored in the springs can be calculated as the sum of the energy stored in the horizontal and vertical springs on Fig. 3:

$$U = \left[ \frac{1}{2} \left( (\rho - h) \cdot \dot{\alpha} \right)^2 + \frac{1}{2} \left( b \cdot \dot{\alpha} \right)^2 \right] \cdot \frac{1}{c_{\text{def}}} \cdot \frac{1}{c_{\text{def}}} \cdot \frac{1}{c_{\text{def}}} \cdot \alpha^2$$

(28)

The elastic energy of a cross-section at $y$ height above ground level:
\[ U = \frac{1}{2} \frac{(y + \rho)^2}{c(y)} \cdot \alpha^2 \]  \hspace{1cm} (29)

Solving Equation 28 and 29 the spring constant \( c_0 \) can be transferred to any cross-section the following way:

\[ c(y) = \frac{c_{def}}{2} \cdot \frac{(y + \rho)^2}{(\rho - h)^2 + b^2} \]  \hspace{1cm} (30)

Replacing \( c_{def} = 4.1 \times 10^{-6} \) m/N measured by Láng in 2003 for a 13 cm trunk diameter cherry tree at 80 cm trunk height, into Equation 30, the change of spring constant along the trunk can be drawn, as shown on Fig. 7.

The \( k(y) \) values can be calculated similarly to the spring constant reduction:

\[ k(y) = \frac{(\rho - h)^2 + b^2}{(y + \rho)^2} \cdot k_{def} \]  \hspace{1cm} (31)

Fig. 8 shows the change of viscous damping coefficient along the trunk using the measured values of Horváth and Sitkei (2002).

\[ \text{Fig. 8. Calculated and measured (●) viscous damping coefficient values along the tree} \]

RESULTS AND DISCUSSION
The actual average idle power values (shaker unclamped, frequency 122 rad/s) with the parameters of the Kilby and Schaumann machines can be calculated in the following form:

\[ P_{r_{\text{ia}}v} = \frac{1}{\pi} \frac{m^2 \cdot r^2 \cdot \omega^3}{2 \cdot (M + m)} \cdot \int_{\alpha = 0}^{\pi} \sin 2 \omega \alpha = 19.7 kW \]  \hspace{1cm} 32.

\[ P_{s_{\text{ia}}v} = \frac{1}{\pi} \frac{m \cdot M \cdot r^2 \cdot \omega^3}{2 \cdot (M + m)} \cdot \int_{\alpha = 0}^{\pi} \sin 2 \omega \alpha = 1.9 kW \]  \hspace{1cm} 33.

The simple tree structure model on Fig. 3 proves to be appropriate to explain the phenomenon of changing virtual turning centre location of the tree when shaking it at
different clamping heights. The reduced mass $M_0$ of a trunk cross-section can be transferred to any other trunk cross-sections. Similarly to the mass reduction, the spring constant and the viscous damping coefficient of a trunk cross-section, based on the principle of energy conservation, can be reduced to any other.

Replacing the three-element model parameters from Fig. 6, 7 and 8 into Equations 14 the average power versus trunk height curves can be drawn. The diagrams generated by a simple PC program are shown in Fig. 9, with unchanged frequency of shaking. It seems clear that the tendency of the curves is influenced both by the machine and tree trunk parameters. The parameters for Equation 14 were taken from the data sheet of a Kilby and of a Schaumann shaker machine.

![Fig. 9. Average power consumption of the two shaker types as function of trunk height](image1)

![Fig. 10. Shaken cross-section amplitude at the two shakers as function of trunk height](image2)

Fig. 10 shows the change of calculated amplitude along the tree trunk, which is influenced by the shaker and tree parameters (Equation 3).

With the free combination of $m$, $r$, $\omega$ and $M$ the amplitude $X$ for the best fruit removal can be set up and the theoretical power needed to drive the harvester machine can be calculated along the trunk ($y$). Note that due to the mechanical and hydraulic losses the real power will be significantly higher.

To judge the efficiency of the shaker types at different shaking heights the specific power (the power needed to generate 1 mm trunk amplitude) can be calculated for each trunk cross-section. As on the Fig. 11 can be seen, when shaking the tree near to soil level the rotating type shaker is more efficient. Clamping the machine higher on the trunk the situation changes: the slider crank type shaker seems to gets more and more efficient.

![Fig. 11. Specific power (kW/mm trunk displacement) needed when shaking with alternating type and rotating type tree shaker](image3)
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Figures

Fig. 1. The model of the tree-shaker system
Fig. 2. Models of the two type inertia shaker: rotating eccentric–weight (a) and a slider crank shaker mechanism (b)

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Fig. 7. Calculated spring constant values along the tree trunk.
Fig. 8. Calculated and measured (*) viscous damping coefficient values along the tree trunk.

Fig. 9. Average power consumption of the two shaker types as function of trunk height.
Fig. 10. Shaken cross-section amplitude at the two shakers as function of trunk height

Fig. 11. Specific power needed when shaking with alternating type and rotating type tree shaker
Modélisation dynamique d'arbre fruitier – Système de Secouage à Inertie

Mots clés : Récolte, équation de puissance, paramètres réduits, amplitude de tronc, fréquence de secouage

Résumé

Les équations de puissance à vide et en charge d'un se coueur avec masse rotative excentrée et d'un du type bielle-manivelle ont été écrites et comparées. Un modèle de la structure, tronc et racines principales, de l'arbre a été construit. En utilisant ses équations statiques, la position du centre virtuel de rotation a été établie en fonction de la hauteur de secouage. A partir de cette fonction, un modèle simple à 3 éléments, composé de la masse réduite, de la constante de raideur et du coefficient d'amortissement cinétique, a été transféré d'une section en coupe de l'arbre à toutes les autres. En reportant ces paramètres réduits dans l'équation de puissance efficace des deux types de secoueur et dans l'équation d'amplitude du tronc, les courbes de puissance efficace consommée et d'amplitude en fonction de la hauteur de secouage ont été tracées. Ces fonctions, avec la libre combinaison des paramètres des secoueurs, excentricité, fréquence de secouage, et masse, facilitent l'optimisation de la conception de la machine. En calculant la relation puissance efficace consommée/ amplitude du tronc pour toutes les sections en coupe du tronc, les hauteurs de préhension les plus efficaces pour les secoueurs ont été déterminées.